

### Marinari et al. reply:

In a comment [1] to our paper [2] H. Bokil, A. Bray, B. Drossel and M. Moore claimed that we have reached wrong conclusions. We show here why their claims are not correct, especially when compared to the analysis of reference [3].

As Bokil et al. correctly point out in [2] we based our analysis on the parameter

$$G = \frac{(\overline{\chi_{SG}^J})^2 - (\overline{\chi_{SG}^J})^2}{V^2 \langle (q - \langle q \rangle)^4 \rangle - (\overline{\chi_{SG}^J})^2}. \quad (1)$$

In the high-temperature phase  $G$  goes to zero like  $V^{-1}$ , where  $V$  is the volume, while it should converge to  $\frac{1}{3}$  in a low temperature replica broken phase [4]. So this new parameter plays the role of the Binder parameter or kurtosis in ordered systems. Obviously, in using  $G$ , one has to be sure that the denominator is non-zero. In one of the cases that we discussed in [2] (finite dimensional Ising spin glasses with two spin interaction) there is no problem since it is well known in the literature that the denominator is non zero [5].

In our paper, instead of presenting the data for  $G$ , we could have shown the data for another relevant parameter ( $A$ ) defined as follows:

$$A = \frac{(\overline{\chi_{SG}^J})^2 - (\overline{\chi_{SG}^J})^2}{(\overline{\chi_{SG}^J})^2}. \quad (2)$$

It is clear that  $A$  does give the same kind of information carried by  $G$ , i.e. it signals the onset of a non self-averaging susceptibility, and does not have the problem of a potentially zero denominator that worries Bokil et al..

In figure (1) we show as an example the results obtained for the four dimensional 3-spin model (the model of figure 4 in reference [2]). The results of figure (1) show unambiguously that the different curves for  $A$ , corresponding to different lattice sizes, cross at  $T = T_c = 2.62$ , and are non-zero below  $T_c$ : in this region  $A$  increases with  $L$ . The parameter  $A$  is a second good indicator for the transition. If  $A$  is not zero, as happens in this case, the numerator and the denominator of equation (1) are both finite in the limit  $L \rightarrow \infty$ .

Although in a complete analysis one should study both parameters, for lack of space it was impossible to present our data for both  $A$  and  $G$ . In our paper we chose to discuss only the parameter  $G$ , since the prediction about its asymptotic value in the low temperature phase (one third) is useful to make the numerical analysis clear, and because the result that the denominator is non zero is so well established (at least in the most important case we were considering).

Now a few words about why we believe that the comment by Bokil et al. is confusing. The reason for which Bokil et al. find a non zero  $G$  in a situation where replica

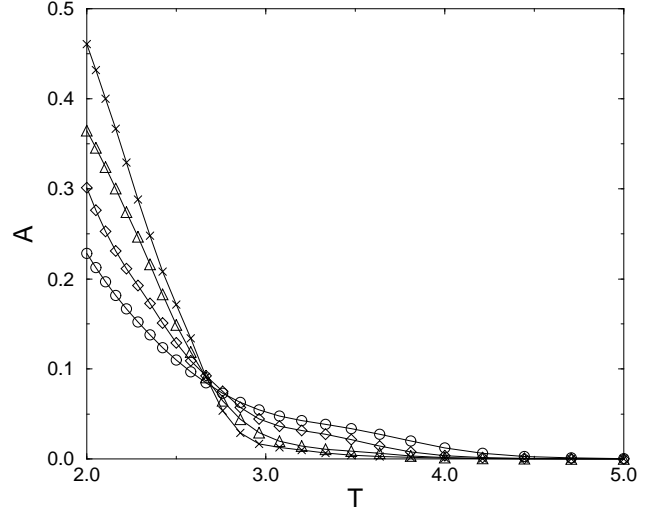


FIG. 1.  $A$  for the 3-spin model.  $L = 3, 4, 5, 6$  correspond to circles, diamonds, triangles and crosses

symmetry is non broken has nothing to do with a small denominator (it is well known that the MK approximation does not describe well spin glasses, since already in mean field it implies a trivial droplet structure [6], failing to describe the rich structure of the SK model).

In three dimensional spin glasses in the MK approximation at  $T = 0.7$ , for reasonable values of  $L$ , the function  $P_L(q)$  looks indeed like a replica broken probability distribution and does not depend on  $L$ . Only for extremely large values of  $L$  one would see that in the MK approximation replica symmetry is not broken: in the MK approximation finite size corrections go to zero extremely slowly. In this situation it is reasonable that any estimator based on the behavior of  $P_L(q)$  will give the same misleading answer for reasonable values of  $L$ . This is what happens for the parameter  $G$  and what likely also happens with the parameter  $A$ .

The test of Bokil et al. only tells us that in the crossover region near the critical temperature where the behavior of the system (in the MK approximation) is dominated by the critical point, the parameter  $G$  takes a value similar to the value at the critical temperature. They left unanswered the most interesting question, which is the asymptotic behavior of  $G$  in a model (like the MK approximation) where replica symmetry is not broken.

There are two separate issues: (a) the existence of strong finite size effects beyond the MK approximation; (b) the necessity of checking that the denominator of eq.

(1) is non-zero. Issue (a) will be carefully discussed in [7] and issue (b) is easily solved complementing  $G$  with  $A$ . Mixing the two issues together, as was done in [1], leads to confusion.

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